on the countries of the Eastern Partnership of the EU and Russia
[2] Forbes / Bitter medicine : how visitors can save the Russian economy http://www.forbes.ru/mneniya-column/idei/246195-gorkoe-lekarstvo-kak-priezzhie-mogut-spasti-rossiiskuyu-ekonomiku
[3] Vorobiova O. D. The migration processes of the population: the theory and the state migration policy // Problems of legal regulation of migration processes in the Russian Federation / analytical compilation of the Federation Council FA RF — 2011. — № 13 (207).
[4] Ioncev V. A. International migration : theory and history of the study II - Edition/ Dialog, / Moscow , 2012
[5] Report of the Secretary-General of the United Nations / International migration and development (from 3.08.2012 years) / 67 session

Oleynikov B.I., Tsylina I.T., Yarnykh E.A.

EVALUATION OF THE EFFECTS ON THE CONSUMER OF INFLATIONARY PRICE CHANGES FOR MULTIPLICATIVE UTILITY FUNCTION

Oleynikov B.I., Russia, Associate Professor of Plekhanov Russian University of Economics
Tsylina I. T., Russia, Associate Professor of Plekhanov Russian University of Economics
Yarnykh E.A., Russia, Professor of Plekhanov Russian University of Economics

Abstract

The effect of inflationary price changes for different goods and services on consumer welfare for multiplicative utility function is evaluated. There were received the resulting estimates of changes in consumer welfare and in the structure of consumer basket in characteristic for inflation simultaneous change in the prices of a large number of benefits.

Keywords: The consumer basket, the vector of prices, consumer welfare, the utility level.

Introduction

In the conditions of inflation price changes of a large number of goods and services occurs within a short period of time. There is an urgent need of finding the value of changes in the welfare of the consumer and quantitative changes in his consumption basket. In this paper the solution to this problem for one type of multiplicative utility function of the consumer is presented.

Materials and methods of research

Let $\bar{X} = (x_1, x_2, \dots x_n)$ be the set of goods of the consumer basket, where $x_i, \ i = 1, 2, \dots n$ is the number of units of the i-th good (goods) consumed, $\bar{P} = (p_1, p_2, \dots p_n)$ is a vector of goods prices, $p_i, \ i = 1, 2, \dots n$ is the unit price of the i-th good. Let the utility function of the consumer has the form:

$$U(\bar{X}) = A \cdot x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdot \dots \cdot x_n^{\alpha_n} = A \cdot \prod_{i=1}^{n} x_i^{\alpha_i}$$

It is easy to show [1, 2] that such a function has all the properties of the utility function. Let the income of the consumer is K money units and it is spent on the purchase of $\bar{X}$ goods.

As is known, the task of the consumer's choice of the optimal set of goods $\bar{X}^* = (x_1^*, x_2^*, \dots, x_n^*)$ reduces to solving the following nonlinear programming problem:

$$U(\bar{X}) \to max$$

$$\begin{cases} (\bar{X} \cdot \bar{P}) \le K \\ \bar{X} \ge 0, \ \bar{P} > 0 \end{cases}$$

or

(1) $$U(\bar{X}) = A \cdot x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdot \dots \cdot x_n^{\alpha_n} \to max$$

(2) $$\begin{cases} x_1p_1 + x_2p_2 + \cdots + x_np_n \le K \\ x_i \ge 0,\ p_i > 0,\ i\quad 1,2,\ldots,n \end{cases}$$

The solution to this problem (denote it by X^*) gives the maximum of the utility function $U(X)$: $\max U(X) \quad U(X^*) \quad U^*$

Let the prices of goods have changed and set to a new vector of prices $P^1 \quad (p_1^1, p_2^1, \ldots, p_n^1)$. In this case, the consumer will have a different optimal set of goods $X^{1*} \quad (x_1^{1*}, x_2^{1*}, \ldots, x_n^{1*})$ and the other maximum value of utility function $max U(X) \quad U(X^{1*}) \quad U^{1*}$, which is also the solution of problem (1), (2) for the new values of prices and costs.

The aim of this work is to determine the amount of change in consumer welfare as a result of price changes, provided that the utility function of the consumer remain unchanged when changing the size of the consumption of each of goods, i.e. $U(X^*) \quad U(X^{1*})$ and the consumer does not lose in utility.

Results and discussion

To save the utility level of consumed goods in case of price change the consumer must change the value of the cost to the value of K^1. The change in welfare is then defined as the difference

$$\Delta K \quad K^1 \quad K$$

and the problem is reduced to finding K^1 and K under condition of $U(X^*) \quad U(X^{1*})$.

The solution of problem (1), (2) can be found by the method of Lagrange multipliers:

- $L(x_1, x_2, \ldots, x_n, \lambda) \quad L(X, \lambda)$ - the Lagrangian function

$$L(X,\lambda) \quad U(x_1, x_2, \ldots, x_n) + \lambda \cdot (x_1p_1 + x_2p_2 + \cdots + x_np_n \quad K)$$

or

$$L(X,\lambda) \quad A \cdot x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdot \ldots \cdot x_n^{\alpha_n} + \\ + \lambda \cdot (x_1p_1 + x_2p_2 + \cdots + x_np_n \quad K)$$

- optimal solution is the solution of the following system of equations:

(3) $$\begin{cases} \frac{\partial L}{\partial x_1} = 0 \\ \frac{\partial L}{\partial x_2} = 0 \\ \dots\dots\dots \\ \frac{\partial L}{\partial x_n} = 0 \\ \frac{\partial L}{\partial \lambda} = 0 \end{cases}$$ or

$$\begin{cases} \frac{\partial U}{\partial x_1} + \lambda p_1 = 0 \\ \frac{\partial U}{\partial x_2} + \lambda p_2 = 0 \\ \dots\dots\dots \\ \frac{\partial U}{\partial x_n} + \lambda p_n = 0 \\ x_1 p_1 + x_2 p_2 + \dots + x_n p_n - K = 0 \end{cases}$$

Denoting ${\partial U}/{\partial x_i} = U'_i$, we obtain that $U'_i = A \cdot \alpha_i \cdot x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdot \ldots \cdot x_i^{\alpha_i - 1} \cdot \ldots \cdot x_n^{\alpha_n} = {\alpha_i U}/{x_i}$, и $U'_i/p_i = {\alpha_i U}/{x_i p_i} = \lambda, (i = 1,2,\ldots,n)$.

Whence it follows that

(4) $\alpha_i / x_i p_i = \alpha_j / x_j p_j$ or $x_i p_i / \alpha_i = x_j p_j / \alpha_j$, $(i, j = 1,2,\ldots,n)$.

Therefore, the optimal solution to the problem is the solution of a set of n systems, each of which contains n linear equations:

(5) $$\left[\begin{array}{c} i = 1,2,\ldots n \\ \begin{cases} \frac{x_i p_i}{\alpha_i} = \frac{x_j p_j}{\alpha_j}, \quad j = 1,2\ldots,n, \quad j \neq i \\ \sum_{i=1}^{n} x_i p_i = K \end{cases} \end{array} \right.$$

This solution gives the optimal values of consumption of the i th good x_i^*, $(i = 1,2,\ldots,n)$ such that $U(X^*) = maxU(X) = U^*$. It is easy to show that the decision of the problem has the form:

(6) $$x_i^* = \frac{K}{p_i} \cdot \frac{\alpha_i}{\sum_{i=1}^{n} \alpha_i}, \quad i = 1,2,\ldots,n$$

(7) $$U^* = A \cdot \frac{K^{(\sum_{i=1}^{n}\alpha_i)}}{\sum_{i=1}^{n}\alpha_i} \cdot \prod_{i=1}^{n}\left(\frac{\alpha_i}{p_i}\right)^{\alpha_i}$$

We assume that when the prices of goods will change the costs to the consumer will also change and will be the value of K^1. In these conditions, the problem of consumer choice is reduced to the solution of problem (1), (2) for new values of prices and costs:

$$\begin{cases} U(X) = A \cdot x_1^{\alpha_1} \cdot x_1^{\alpha_2} \cdot \ldots \cdot x_1^{\alpha_n} \to max \\ x_1 p_1^1 + x_2 p_2^1 + \cdots + x_n p_n^1 \le K^1 \\ x_i \ge 0, \; p_i^1 \ge 0, \; i = 1, 2, \ldots n \end{cases}$$

The solution to this problem is similar to the previous one and gives the following result:

(8) $$x_i^{1*} = \frac{K^1}{p_i^1} \cdot \frac{\alpha_i}{\sum_{i=1}^{n}\alpha_i}, \quad i = 1, 2, \ldots, n$$

(9) $$U^{1*} = A \cdot \frac{(K^1)^{(\sum_{i=1}^{n}\alpha_i)}}{\sum_{i=1}^{n}\alpha_i} \cdot \prod_{i=1}^{n}\left(\frac{\alpha_i}{p_i^1}\right)^{\alpha_i}$$

Using (9), from the condition $U^* = U^{1*}$ we find K^1 and, therefore, we find ΔK.

Conclusions

(10) $$K^1 = K \cdot \prod_{i=1}^{n}\left(\frac{p_i^1}{p_i}\right)^{\frac{\alpha_i}{\sum_{i=1}^{n}\alpha_i}}$$

(11) $$\Delta K = K \cdot \left[\prod_{i=1}^{n}\left(\frac{p_i^1}{p_i}\right)^{\frac{\alpha_i}{\sum_{i=1}^{n}\alpha_i}} - 1\right]$$

$$= K \cdot \left[\left(\frac{p_1^1}{p_1}\right)^{\frac{\alpha_1}{\sum_{i=1}^{n}\alpha_i}} \cdot \left(\frac{p_2^1}{p_2}\right)^{\frac{\alpha_2}{\sum_{i=1}^{n}\alpha_i}} \cdot \ldots \cdot \left(\frac{p_n^1}{p_n}\right)^{\frac{\alpha_n}{\sum_{i=1}^{n}\alpha_i}} - 1\right].$$

Let $\left(\frac{p_i^1}{p_i}\right) = z_i$ be the coefficient of change of the price of the $i-th$ good

$$z_i = \frac{p_i + \Delta p_i}{p_i} = 1 + \frac{\Delta p_i}{p_i} = 1 + \delta p_i$$

where Δp_i is the absolute, δp_i is the relative price change. It is obvious that for the *i*-th good the coefficient $z_i = 1$ if the price of this good has not changed, $z_i > 1$ at higher prices, $z_i < 1$ if price goes down. Note that if you change the consumer welfare ($\Delta K \neq 0$), the consumption of those goods for which the price has not changed will also change. The amount of change of the consumption level of the i-th good Δx_i^* is obtained from (8) and (10):

$$(12) \quad \Delta x_i^* = x_i^{1*} - x_i^* = \left(\frac{K^1}{p_i^1} - \frac{K}{p_i}\right) \cdot \frac{\alpha_i}{\sum_{i=1}^{n} \alpha_i} = K \cdot \left(\frac{1 + \Delta K / K}{p_i^1} - \frac{1}{p_i}\right) \cdot \frac{\alpha_i}{\sum_{i=1}^{n} \alpha_i}$$

The expression (13) can be written in the form:

$$(13) \quad \Delta K = K \cdot (Z - 1)$$

From (15) it follows that at $Z \le 1$ consumer welfare is not impaired, at $Z < 1$ it is improved; and when Z>1 the consumer's welfare is deteriorating because to save the utility level after the price change he is forced to spend on the goods an amount greater by the amount $\Delta K > 0$.

Example. Let $U = 4x_1^{1/2} x_2^{1/3} x_3^{1/4} x_4^{2/7}$ be the utility function of the consumer. The prices of each of the four types of goods are: $p_1 = 12$ money units, $p_2 = 9$ money units, $p_3 = 8$ money units, $p_4 = 7$ money units. The price of the first type of goods decreased by 3 money units, of the second and third - increased by 2 units, the forth has not changed. How will the consumer's welfare change when prices change if the consumer decided not to change the level of utility of consumption goods? What types of goods have changed the consumption when prices change and how much, if prior to the change of prices the consumer spent 2000 money units at all kinds of goods?

Results. In accordance with the expressions (13) and (15) we have:

$$\Delta K \quad K \cdot \left| \left(\frac{p_1^1}{p_1}\right)^{\frac{\alpha_1}{\alpha_1 \; \alpha_2 + \alpha_3 + \alpha_4}} \cdot \left(\frac{p_2^1}{p_2}\right)^{\frac{\alpha_2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}} \cdot \left(\frac{p_3^1}{p_3}\right)^{\frac{\alpha_3}{\alpha_1 + \alpha_2 \; \alpha_3 + \alpha_4}} \cdot \right.$$

$$\left. \cdot \left(\frac{p_4^1}{p_4}\right)^{\frac{\alpha_4}{\alpha_1 \; \alpha_2 + \alpha_3 + \alpha_4}} \quad 1 \right|$$

$$K \cdot \left| \left(\frac{9}{12}\right)^{\frac{1,115}{2/84}} \cdot \left(\frac{11}{9}\right)^{\frac{1,115}{3/84}} \cdot \left(\frac{10}{8}\right)^{\frac{1,115}{4/84}} \cdot 1 \quad 1 \right|$$

$$0{,}015K \quad 30$$

As a result of price changes the consumer's welfare will not worsen, but rather, he will win in his budget 30 money units or 1.5 percent of the previous cost.

Formula (14) for $K \quad 2000$ and $K^1 \quad 1970$ shows that consumption of the first type of goods will increase by 19 units, second, third and fourth – will be decreased, respectively, by10,5; 9,7 and 0,9 units.

References

[1.] Ashmanov S. A. Introduction to mathematical Economics. – M.: Nauka, 1984.

[2.] Zamkov O. O., Tolstopyatenko V. A., Cheremnykh Y.N. Mathematical methods in Economics. – M.: Moscow state University, 2001.