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ON THE MATHEMATICAL MODELS OF POSSIBLE FORMS OF INTERACTIONS OF THE BRUSH-COMMUTATOR UNIT ELEMENTS

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Abstract

The dynamics of the brush-commutator unit as a factor predetermining switching of the electric traction motor of the electric locomotive is considered in the article. The subject of the study is the characteristics of motion with a gap – a dynamic effect that manifests itself as the velocities and amplitudes of the oscillations of the contacting surfaces of the elements of the brush-commutator unit of the traction motor increase.

The analysis is carried out using a model problem in which the brush, denoted by a material point, is being considered in accordance with the rules of periodic motions with continuous tossing on the vibrating surface and the possibilities of implementing vibro-impact modes.

The possible detachment is determined by the influence of force factors, which can be characterized by pressing the brush to the commutator with the brush holder, as well as by external force factors caused by the vibrations of the traction motor.

Unilateral constraints are taken into consideration using the generalized gap function method.

The article provides analytical results of the study of the interaction features in the "brush-commutator" contact.

A generalized approach is developed for the problems of estimating the dynamic interactions of the elements of the brush-commutator unit implemented in the modes of continuous contact failure.

The basis of the approach is the introduction of the concept of gap function, which makes it possible to justify the construction of the brush motions in accordance with certain criteria that reflect the properties of the brush motion forms in interaction with the commutator surface.

Diagrams of the forms of motion of the brush-commutator unit elements are presented on the basis of numerical modeling results.

Keywords: traction electric motor of electric locomotive, brush-commutator unit, brush-commutator interaction, unilateral constraints, gap function

Introduction. While ensuring the operational reliability of high-speed and heavy haul rolling stock, high emphasis is placed on the dynamic quality of the brush-commutator unit.

A number of fundamental works [1-3] are focused on the evaluation of the influence of mechanical factors on the switching process.

Depending on the characteristics of the dynamic loads, the switching efficiency reflects the peculiarities of contacting the brushes, taking into account the unilateral nature of the constraints.

Such a dynamic effect as a motion with a gap manifests itself with increasing velocities and amplitudes of oscillation of the contacting surfaces of the elements of the brush-commutator unit of the traction motor.

Studies in the field of rolling stock dynamics have shown that unilateral constraints that allow disruption of the continuity of the interaction characteristics between the brush and the commutator can have a significant effect on the operation of traction motors [4].

This is due to the fact that during the current pick-up and commutation in the brush-commutator unit of the traction motor (ETM), a periodic failure of the brush-commutator contact creates an undesirable process [5].

The problems of taking account of unilateral constraints as a factor of dynamic effects were repeatedly raised in papers on the theory of vibratory displacement [6].

A number of problems reflecting unilateral interactions of material points with vibrating surfaces were considered in studies on the justification of the use of vibrational technologies for the mining industry, the construction industry, engineering and transport [7-9].

To model the features of the unilateral interaction of a material point with a vibrating surface, the approach involving the gap function [9] was used in the works on the dynamics of the vibratory technological machines. This represents an analytical apparatus for determining the detailed characteristics of the forms of motion of elements taking account of the unilateral constraints.

Despite considerable progress in the study of dynamic effects arising due to the presence of unilateral constraints, many problems on the dynamics of the elements of the brush-commutator unit (BCU) have not been sufficiently elaborated in terms of the dependence on the nature of the constraints in the "brush-commutator" contact.

This article is focused on the approach to constructing a mathematical model on the basis of a generalized method taking into consideration unilateral constraints when applied to the dynamics of the BCU elements.

I. Basic provisions. Problem formulation.

With all the significance of the notion that the brush is a solid, to determine the conditions of interaction between the brush and the commutator, if we do not take into account the dimensions of the brush and confine ourselves to the consideration of vertical oscillations (or vibrations), the computational scheme can generally be a material point, which is located on a vibrating surface.

There is a bond between the brush and the commutator surface and a reaction occurs between them; at certain parameters of the surface motion, the bond can be broken, then the brush can perform an autonomous motion, and a gap appears with the subsequent restoration of the contact.

The operation of the brush contact is determined not only by the vibrations of the contact surface. The connection between the brush assembly and the body of the ETM, which operates under conditions of intense dynamic loading, has a direct effect on the dynamic interaction of the brush with the commutator.

In traction engines of electric locomotives, the brushes are spring-loaded and located around the commutator along the perimeter.

The loading conditions for the brush contact are different, which is reflected in the computational schemes of the brush-commutator unit (BCU).

As the basic model, the vertical arrangement (vertical assembly) of brushes is considered due the assumption that it is possible to take into account the peculiarities of the arrangement of brushes and contact properties by appropriate adjustments of the basic model.

If there are any irregularities and deviations from the cylindrical shape, the rotating commutator is interpreted on the basis of the inversion principle as a certain oscillating plane [10] (Fig. 1).

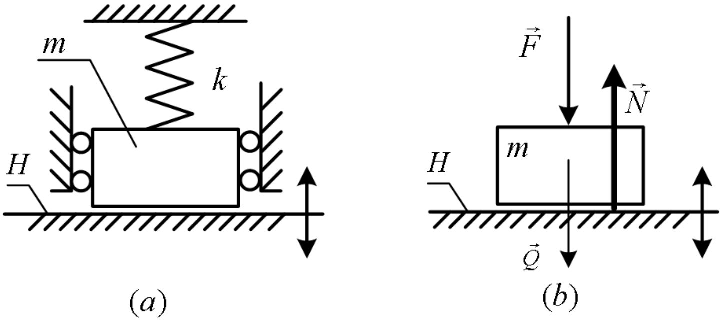

Figure 1. Brushing the brush to the commutator: (a) - contact diagram, (b) - the design scheme of the brush and commutator interaction in the contact phase

Figure 1 shows the surface of the commutator *H* and the brush *m*. In general, the force of gravity *Q* acts on the brush, the additional force *F* (the spring pressing force, as well as other forces caused by motions of the ETM body).

The surface performs harmonic vertical motions $H(t) = A\sin(\omega t)$. At certain values of the amplitude *A* and frequency ω, the brush *m* acquires the possibility of detachment from the surface of the commutator – a free approach with a subsequent contact or impact (as a form of the contact restoration).

Under certain conditions, stable regimes can arise with the brush being detached from the commutator [9].

The task is to develop a generalized method for determining the key dynamic characteristics that reflect the features of the brush motion patterns in the unilateral interaction with the commutator.

II. Forms of brush motion taking into account unilateral constraints in interaction with the commutator. Features of the mathematical model.

To consider the motion of the brush, we introduce the form *X(t)*, which is a graph of brush motion without contact with the commutator, except for the moment of detachment t_0 .

At a preliminary stage, we believe that there are no other forces than gravity force.

In the approach phase, the form *X(t)* satisfies a system of second-order differential equations with initial conditions determined at the moment of motion t_0 of the commutator surface:

$$\begin{cases} \ddot{X}(t) = -g, t \ge t_0 \\ \dot{X}(t_0) = \dot{H}(t_0) \\ X(t_0) = H(t_0) \end{cases} . \quad (1)$$

The motion of the brush (with tearing capabilities) consists of alternating phases of approach and contact.

The approach phase are the moments when the brush moves without contact with the commutator surface, in this case the motion satisfies the equation (1); the phase of the contact consists of the moments when the brush moves in contact with the commutator, in which case the brush moves according to a harmonic law, reproducing, under the assumptions, the shape of the motion of the surface *H(t)*.

The shape of the brush motion with the alternation of the contact and approach intervals above the commutator surface is shown in Fig.

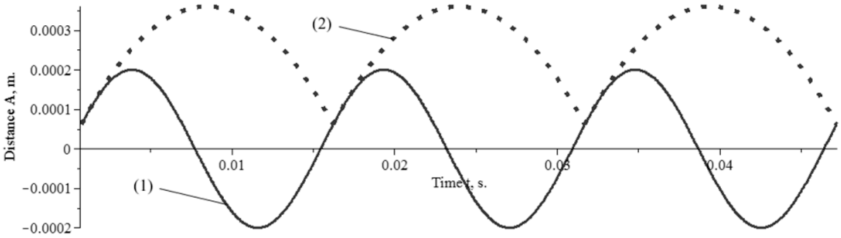

Figure 2. Typical form of tossing brush motion: 2 is the brush motion shape, 1 is the harmonic form of commutator surface motion with amplitude A = 0.0002 m and frequency ω = 64 Hz.

The main characteristics of the interaction between the brush and the surface of the commutator include the conditions of detachment, the moment of detachment, the height of the approach of the brush in the process of autonomous motion, and periodic modes [9].

Of interest are one-touch periodic modes in, when the moment of detachment and the features of the autonomous motion of the brush result, at the moment of the contact restoration, in the same conditions that caused the initial detachment and further autonomous brush motion.

Such modes can be classified by the multiplicity criterion – the number of periods of oscillation of the commutator surface that constitute the period of autonomous brush motion without contact with the commutator.

The internal and external forces arising during the motion of the elements of the brush-commutator unit determine the dynamics of interaction between the brush and the commutator.

III. The calculation of the force pressing the brush holder on the brush.

The action of the pressing force on the brush holder side, assuming that the brush makes small oscillations, can be reflected in the mathematical model with the help of an additional force *F*.

The conditions of preliminary deformation of the clamping spring can be interpreted in some cases in the form *F* □ const.

In the process of dynamic loads generated by the vibrations of the engine body, the force *F* can change its direction in vertical motions.

With regard to the loading of brushes operating in inclined positions and at a lower location, these cases can be considered on a general methodological basis.

Thus, the force *F* can act both in the same direction with the force *Q*, and in the opposite direction.

In addition, we will assume that it is possible to switch the direction and modulus of the force *F* at the point of maximum approach of the brush.

Such situations arise when the brush is detached in a complex force field.

Figure 3 shows the computational scheme of the forces *Q* and *F* under consideration acting in the same direction.

Figure 3, *a* shows the computational scheme for the gap between the brush and the commutator.

Figure 3, *b* shows the computational scheme for the contact interval with the commutator surface, where *N* is the normal reaction at the point of contact.

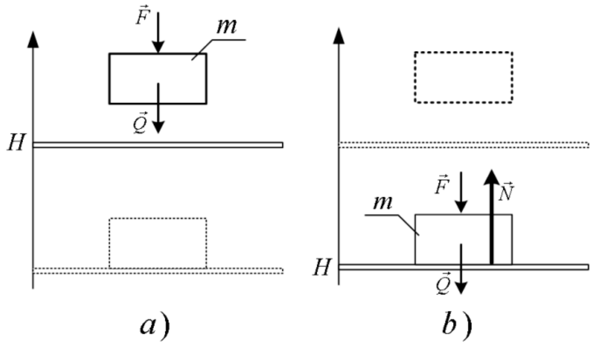

Figure 3. Brush motion taking into account the forces *Q* and *F* acting in one direction: *a* is the approach span, *b* is the contact interval

To determine the forces acting on the brush in the interval of its contact with the commutator, the d'Alembert principle can be applied. If *X(t)* is the shape of brush motion during its contact with the commutator surface, then the following condition is fulfilled:

$$-m\ddot{X}(t)+\vec{N}+\vec{Q}+\vec{F}=0 \tag{2}$$

Brush removal from the commutator leads to the formation of a gap – the positive distance between the brush and the vibrating surface of the commutator at a certain time.

VI. Development of an approach based on the consideration of common forms of brush motion.

Let us denote the shape of the brush motion by *X*(*t*). We fix the definite amplitude *A* and the frequency ω of the oscillation of the surface *H(t)* characterizing the oscillation of the commutator.

We assume that at the moment of detachment t_0, the brush is in such contact with the commutator that the speed of the brush coincides with the velocity of the commutator surface.

If t_0 is the moment of brush detachment from the commutator, then for some time interval starting from t_0 the shape of the brush motion satisfies the system of equations:

$$\begin{cases} \dfrac{\partial^2 X_H(t,t_0)}{\partial t^2} = -g, t \ge t_0 \\ \left.\dfrac{\partial X_H(t,t_0)}{\partial t}\right|_{t=t_0} = \omega A\cos(\omega t_0) \\ \left. X_H(t,t_0)\right|_{t=t_0} = A\sin(\omega t_0), \end{cases} \tag{3}$$

where $X_H(t,t_0)$, which is regarded as a function of t, is the shape of the brush motion.

It should be noted that the detachment of the brush from the commutator is not implemented every instant of time t_0 .

At the same time, for arbitrary t_0, we can formally consider the system of equations (3) and obtain a solution on the interval (t_0,∞). Suppose that for an arbitrary t_0 the solution of system (3) has the form $X_H(t,t_0)$. Considering t_0 as a parameter, we obtain a set of forms of brush motion $X_H(t,t_0)$ at the time interval $t \ge t_0$. Many forms of the brush motion are determined by the motion of the commutator surface *H(t)*. Namely, each shape of the brush motion is uniquely determined by its displacement and velocity at time t_0, which informs them of the commutator surface in vertical motion. The forms of brush motion, which are the solution of the system (3), have an analytical form:

$$X_H(t,t_0) = A\sin(\omega t_0) + A\omega(t-t_0)\cos(\omega t_0) - \frac{1}{2}g(t-t_0)^2. \quad (4)$$

Thus, the shape of the brush motion will consist of alternating fragments of the forms XH (t, t0) (4) when the brush is detached from the surface and fragments when the brush is in contact with the surface of the commutator.

Figure 4 shows the shape of the brush motion, the initial conditions of which are determined by the oscillation of the commutator surface.

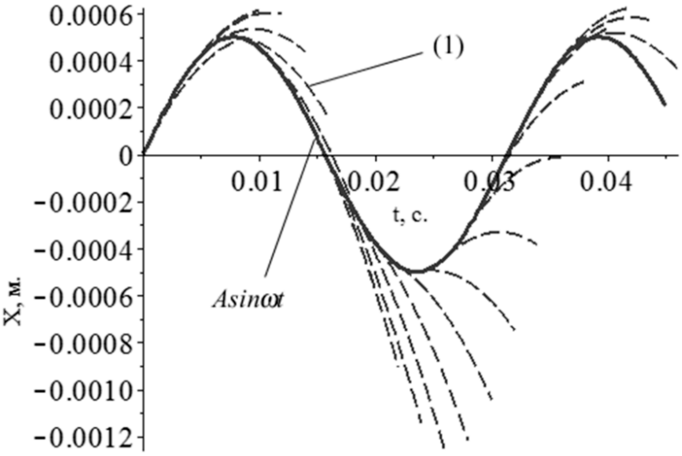

Figure 4. Shapes of brush motion determined by the initial conditions of the commutator oscillation: 1 - brush shape, A = 0.0005 m, ω = 200 rad / s.

For the brush detachment, it is necessary and sufficient for the brush and commutator motion graphs to have only one common point of contact: the detachment point.

Determining the moment of detachment of the brush from the commutator can be made by using the selection criterion among the forms of motion with all possible initial conditions permitted by the motion of the commutator, the forms resulting in the occurrence of a positive distance between the brush and the commutator.

VI. Determining the peculiarities of brush motion forms on the basis of the generalized gap function method.

The theoretical basics of the gap function method are presented in [9]. In order to interpret the method within the scope of the indicated problem, we consider the function $X_H(t,t_0)$ – a family of brush motion forms with initial conditions allowed by the oscillations of the commutator surface. The detachment of the brush at the time point can be interpreted as a condition $X_H(t,t_0) > H(t)$ performed in a small time interval after the detachment moment, provided that $X_H(t_0,t_0) = H(t_0)$. Until the time of detachment, the brush can either be in contact with the commutator surface, or be in the approach. This means that the backgrounds of the detachments are different.

The process of the brush motion in contact with the commutator is called the brush idling. If the detachment occurs after the idling interval, then such a detachment is considered a detachment with a preliminary idling of the brush.

The value of the idling time interval can be different. There are two possible options.

First, the brush is constantly in contact with the commutator. The second is that the contact time interval degenerates into a point. In the second case, the detachment is implemented without the preliminary idling.

The characteristic of brush detachment from the commutator is the difference in the position of the brush and the surface of the commutator. Time function:

$$R(t) = X(t) - H(t) \tag{5}$$

shall be called the clearance between the brush and the commutator surface.

To obtain the detachment criterion, the gap function is introduced into consideration.

The gap function, which depends on two instants of time (t,t_0), such that $t \geq t_0$ represents the difference $R_H(t,t_0) = X_H(t,t_0) - H(t)$. In the case of the harmonic law of motion of the commutator surface and the presence of only the forces of weight acting on the brush, the gap function has the form (4).

On the basis of the gap function, various selection criteria for the detachment forms can be constructed. An example is a differential criterion based on the use of derivatives of the gap function to determine the detachment time.

The moments of detachment, in turn, can be classified on the basis of various characteristics.

Thus, the detachment moments obtained on the basis of a differential criterion can be classified on the basis of the order of the derivative.

It was shown in [9] that the detachment times can be divided into detachment points of the second and third order.

The order of detachment can be interpreted as the "smoothness" of detachment.

Table 1 presents the necessary and sufficient conditions for the detachment of the second and third order, obtained on the basis of a differential criterion.

Table 1. Conditions of detachment

The order of detachment	Analytical conditions
0	Does not exist
1	Does not exist
2	$\frac{A\omega^2}{g}\sin(\omega t) > 1$
3	$\begin{cases} \frac{A\omega^2}{g}\sin(\omega t) = 1 \\ \cos(\omega t) > 0 \end{cases}$
>3	Does not exist

Examples of forms of motion of the brush and the commutator surface with different detachment points are shown in Figure 5.

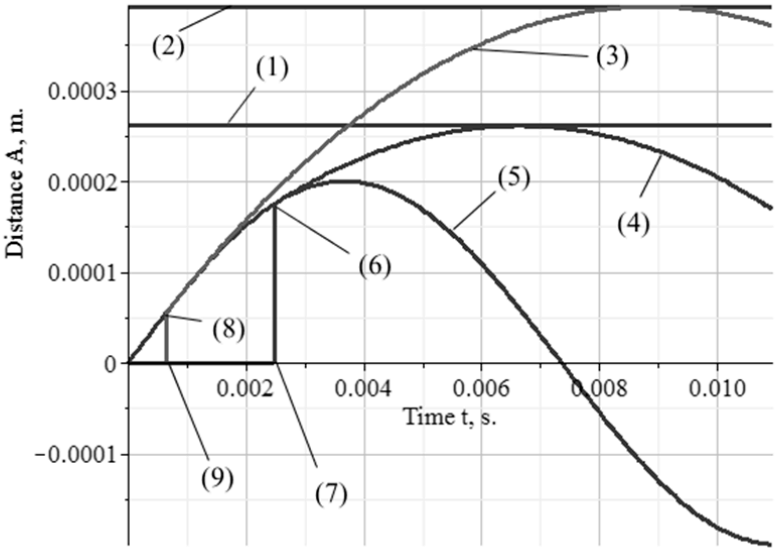

Figure 5. The shape of the brush motion from the detachment points of the second and third order: A = 0.0002 m, ω = 46 Hz., *5* is the surface of the oscillation of the commutator, *9* is the time of the detachment time of the form *3* at the point *8* of the detachment of the third order, *7* is the detachment of form *4* at the point of detachment *6* of the second order, *1*, *2* are levels of the maximum approach of the brush

The detachment criterion makes it possible to determine the detachment point and a number of basic indicators of the form of motion in an analytical form, in particular, to establish a connection between the nature of the position and the detachment order.

Conclusions.

As a result of the studies of the dynamics of the brush-commutator unit, the following conclusions can be drawn:

1. A generalized approach is developed for the problems of estimating the dynamic interactions of the elements of the brush-commutator unit implemented in the modes of continuous contact failure. The basis of the approach is the introduction of the concept of a gap function, which makes it possible to justify the construction of the brush motions in accordance with certain criteria that reflect the properties of the brush motion forms in interaction with the commutator surface.

2. The features of the dynamic interactions of the brush and the commutator in the process of the brush-commutator unit, in which the effects of unilateral constraints are manifested, have been studied; analytical approaches have been defined and developed in the assessment of the possibilities of vibrational interactions of the brush and commutator with a continuous tossing.

3. Mathematical models of the formation of vibrational interactions applied in the problems of the dynamics of the brush-commutator unit are developed taking into account factors of technological importance or criteria, which include the conditions for the implementation of divisible modes and the effect of additional forces.

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